

From the electric field E graph it is obvious that the field lines are perpendicular to the edges. In general we expect the at the edges to have circular equipotentials but here they have a different shape and this is because the field at the edges is grounded so between the edges and the line charge an electric field is created due to the voltage difference and also because we do not have an infinite distance from the charge line but instead we have limited it to the square edges from the graph as we have written our code to work like that.



As we know from the lecture the magnitude E of the electric field is inversely proportional to the distance of the line charge. So at the centre the distance from the line charge is 0 so 1/r goes to infinity and so E tends to infinity and vice versa at the points that are far away from the centre we can se that E tends to zero.



These two graphs are the same with the two previous ones and that's because Er = 1.



As it is mentioned before t magnitude E of the electric field is proportional to the distance of the line charge 1/r. So from this, all the points that have the same distance from the charge line must be on the same equipotential. At this task this can easily be observed from the graphs and this is due to the cylindrical enclosure that makes the equipotential lines to be circular. On the other hand, at task 1, there are points that have the same distance from the centre (the line charge) but the Electric field is different and this is because the square enclosure.



These graphs look like the some graphs from task one with a small difference in magnitude towards the origin.



The above is the log-log plot of the field along the x-axis. The graph is almost a straight line with negative slope, as expected. The slope can be easily calculated:

$$m = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{x_2}{x_1}\right)} = \frac{\log\left(\frac{363}{4124}\right)}{\log\left(\frac{10}{1}\right)} = -1.09$$

The above means that the electric field along the x axis is $E_x \propto \frac{1}{r}$





From the graphs we can see that as the distance from the dielectric becomes bigger the equipotential and the field lines becomes parallel and this fact help us to come to the conclusion that if the dielectric constant was the same as before (Er = 1) then the we would not have variation at the field. An important thing to be made clear is that a dielectric put inside an electric field creates a uniform electric field of its own. This is because the charged particles of the dielectric are attracted at the edges of the dielectric. Specifically, electrons are attracted towards the positive rail at the top and protons are attracted towards the negative rail at the bottom. This has the effect of creating separated charges in the dielectric which creates an electric field. This field has the opposite direction and that results to the total magnitude of the field inside the dielectric to be less than the magnitude out of the dielectric



On the other hand the flux density obeys the equation:

$$D = \varepsilon_r * E$$

In this case $\varepsilon_r = 4$ so we expect the values of the flux to be 4 times bigger than the field inside the dielectric which is true.

Running the code but with grounded left and right sides gives the field lines and equipotential lines shown below:



This is again a similar effect as the one seen in Task 1. Because the edges of the enclosure simulate infinite distance, having a fixed potential at 0 there results in a field being created between the grounded enclosure and the plates. If we extended the calculation window, the effects at the edges would remain the same, but we could clearly see normal behaviour at a range close to the middle of our window. This in fact is a better way of approximating infinity as we are actually increasing the space of observation for our experiment.



Now we have two wire electrodes held at potentials of ± 10 V and separated by a distance of 0.4 x SPAN, mounted symmetrically on the x-axis in a square grounded enclosure.



From potential φ graph: as the distance from the charges increase the magnitude of Vo decreases and also because the voltages of the two charges are opposite between them the equipotentials are denser, meaning that between the charges the field is bigger.

From Electric field E graph: between the two charges the electric field lines are more dense and again as task 1 the field at the edges is grounded so between the edges and the line charge an electric field is created due to the voltage difference and also because we do not have an infinite distance from the charge line but instead we have limited it to the square edges (field lines are perpendicular to the edges).



So here we can observe that at zero distance to the charges E tends to infinity and vice versa at the points that are far away from the charges we can see that E tends to zero and that is because as the distance from the charges becomes bigger the field becomes smaller and smaller(that is because as I mentioned before E is inversely proportional to the distance r).

An important graph to explain is the one that shows the field along the vertical central line. Because at this line the distance between the charges is the same in magnitude and because the charges are opposite, the y components of the field cancel out giving a 0 y component at every point on the line. The x components of the electric field of the charges, on the other hand, add up giving a peak magnitude at y=0 because at that point the distance between both charges is minimised. Away from that point the magnitude of the field decreases close to 0.

Task 5 The extra lines of code are shown below:

case 5

VF=10; EP(CEN-0.2*SPAN:CEN+0.2*SPAN, :)=4; V0(1:CEN-0.2*SPAN,:)=-VF; F(1:CEN-0.2*SPAN,:)=0; V0(CEN+0.2*SPAN:IMAX, :)=+VF; F(CEN+0.2*SPAN:IMAX, :)=0;



From fixed potential φo we can see the capacitor plates and the dielectric between them



It is clear, that between the plates the field does not vary, in other words it is uniform.



Along the x axis. There is only a y component of the field, This is because the field is uniform and has direction from the top capacitor plate to the bottom(i.e. it is parallel to the y axis). Not how along the x axis the field component is constant which verifies that the field is uniform. Also along the y axis the y component of the field is zero for any point not inbetween the capacitor plates, and it has a constant value between them with the same magnitude as along the x axis as expected.



As the field has no x component we verify this from the field and flux graphs where the x component is 0 for all values of distance. The only varying component is the y component which has a constant value between the capacitors. The value of the flux between the capacitors can be interpreted using the boundary condition between two domains as the surface charge on the boundary of the domains(i.e. the surface charge of the capacitor plates).

```
The extra lines of code are shown below:

case 6
VF=+10;
EP(CEN-0.2*SPAN:CEN+0.2*SPAN, :)=4;
V0(CEN-0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=-10;
F(CEN-0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=0;
V0(CEN+0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=+10;
F(CEN+0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=0;
```

```
V0(IMAX, :) = 0;
F(IMAX, :) = 0;
V0(1,:) = 0;
F(1,:) = 0;
V0(:,IMAX)=0;
F(:,IMAX)=0;
V0(:, 1)=0;
F(:, 1)=0;
```



The electrodes are separated by a continuous dielectric slab of relative dielectric constant $\varepsilon_r = 4$, mounted symmetrically in a square grounded enclosure to span its whole width. Moreover, the plates of the capacitor are infinite so we can say that this statement is true for the dielectric too.



As we can see between the plates of the capacitor the electric field is uniform and the lines are parallel. For one more time the field is mounted symmetrically in a square grounded enclosure so at the edges we can observe a distorting effect(between the edges and the line charge an electric field is created due to the voltage difference). Having a grounded anclosure acts as a fixed potential in space which creates an electric field with the plates.



A nice observation to be made is that along he horizontal CL the field has decayed to a magnitude of 0 whereas for the vertical CL there is a y component even at the edges of the space. This means that the field is stronger along the y axis.



The extra lines of code are shown below:

```
case 7
VF=10;
EP(CEN-0.2*SPAN:CEN-1, :)=3;
EP(CEN+1:CEN+0.2*SPAN, :)=6;
EP(CEN, :)=4.5;
V0(CEN-0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=-10;
F(CEN+0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=0;
V0(CEN+0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=+10;
F(CEN+0.2*SPAN,CEN-0.4*SPAN:CEN+0.4*SPAN)=0;
```





The graph shows a finite capacitor, with horizontal sheet electrodes of width 0.8 x SPAN, held at potentials of ±10 V. Half of the slab thickness of the dielectric slab consists of material of dielectric constant $\varepsilon_r = 3$ and the other half of material with $\varepsilon_r = 6$ (and to smooth things out we use $\varepsilon_r = 4.5$ at y=0).



The important thing to note on the graph of the electric field is that it is stronger in the part of the dielectric with er = 3. We would expect a magnitude double the size of the one in the

dielectric slab of er = 6 and as it is seen in the field along the horizontal CL it is indeed nearly double.



The field along the vertical CL makes an immediate change when crossing from one dielectric to the other. Because Matlab solves this problem numerically and not analytically this corresponds to a kink in the flux density graph which is due to the limited resolution of our space where that immediate change can not be described continuously. Other than the kinks of the flux graph, the flux satisfies the boundary conditions which imply continuous flux between boundaries.



The extra lines of code are shown below:

case 8
VF=+10;
EP(CEN-0.2*SPAN:CEN+0.2*SPAN, :)=4;
V0(CEN+0.2*SPAN,CEN-0.25*SPAN:CEN+0.25*SPAN)=+10;
F(CEN+0.2*SPAN,CEN-0.25*SPAN:CEN+0.25*SPAN)=0;
V0(1:CEN-0.2*SPAN,:)=-VF;
F(1:CEN-0.2*SPAN,:)=0;

```
V0(IMAX, :) = 0;
F(IMAX, :) = 0;
V0(1,:) = 0;
F(1,:) = 0;
V0(:,IMAX)=0;
F(:,IMAX)=0;
V0(:, 1)=0;
F(:, 1)=0;
```







The important thing to note on this task is the near uniformity near the center of x axis. The fields created are between the strip electrode and the horizontal strip electrode, and these two with the grounded enclosure. The field is not entirely uniform because there is a substantial field component created by the interaction with the grounded enclosure. To minimise this effect, the potential of the horizontal sheet electrode could be made much larger so that the component due to the grounded enclosure is minimised.



There are many irregularities in the field graphs, which imply a strong effect of the grounded enclosure to our setup. Along the x axis the x component of the field is mostly a product of the interaction of the microstrip waveguide with the grounded enclosure. We want a near 0 magnitude on the edges of space and we have considerable values. This is true as well for the y component of the field where it has a peak on the edges of the graph. Other than that, the y component has its peak between the strip electrode as expected.



Task 9 These are the extra lines of code for example 9.

V0(CEN+0.2*SPAN,CEN-round(0.25*SPAN):CEN+round(0.25*SPAN))=+VF; F(CEN+0.2*SPAN,CEN-round(0.25*SPAN):CEN+round(0.25*SPAN))=0; V0(CEN-1, :)=0; F(CEN-1, :)=0;

V0(IMAX, :) = 0; F(IMAX, :) = 0; V0(1,:) = 0; F(1,:) = 0; V0(:,IMAX)=0; F(:,IMAX)=0; V0(:, 1)=0; F(:, 1)=0;





The field components are similar with the ones from task 8. The big difference is on the magnitudes of the field components rather than the shape of the curves which is due to the decreased separation of the strip and the horizontal sheet while also grounding the sheet. Note how the grounded enclosure does not affect the field components as in task 8 which leads to more symmetric components that are closer to being uniform between the electrode strip and the horizontal sheet.

e-10
c =
e-10
.c

By integrating the flux component along the x axis one obtains the stored charge in terms of the dielectric constant multiplied by the permittivity of free space. To obtain the actual charge we need to multiply the result by this multiplication product. Then the capacitance is just the charge divided by the fixed potential of 10V.

To get the simple analytic expression of the capacitance in terms of the separation of the strip and the width of the strip, we just divide the width with the separation and multiply the result with the product dielectric constant and the permittivity of free space.

MAGNETIC Task 1

By dividing equation 2.5 with μ_0 we get and taking *a* that tends to infinity:

$$H_{y} = \left(\frac{I}{4\pi}\right) \left(\frac{z}{y^{2} + z^{2}}\right) \left(\frac{(x-a)}{\{(x-a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}} - \frac{(x+a)}{\{(x+a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$
$$= -2\left(\frac{I}{4\pi}\right) \left(\frac{z}{y^{2} + z^{2}}\right)$$
$$H_{z} = \left(\frac{I}{4\pi}\right) \left(\frac{-y}{y^{2} + z^{2}}\right) \left(\frac{(x-a)}{\{(x-a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}} - \frac{(x+a)}{\{(x+a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$
$$= -2\left(\frac{I}{4\pi}\right) \left(\frac{-y}{y^{2} + z^{2}}\right)$$



For an infinite wire carrying current of 1A we get the magnetic field components shown below:





Indeed the field components generated by the MatLab code are closely related to the analytical expressions derived and also note the inverse sign between the two expressions and how it is depicted in the field components.

Task 2 For 2 wires we have:



It is important to analyse how the field components vary along the y axis. For the range - 50 < y < 20 the dominant wire is the leftmost wire and that is the reason that the z component of the field resembles the field of one current carrying wire. For the range of -20 < y < 20 both of the wires affect the field component in equal weight which is the superimposed version of their individual fields.



Indeed, the analytical expression closely resembles the generated field components of the Matlab code by plotting both variations in Excel.

Task 3 By using equation 2.5 but taking a small *a* we get:

$$H_{y} = \left(\frac{l}{4\pi}\right) \left(\frac{z}{y^{2} + z^{2}}\right) \left(\frac{(x-a)}{\{(x-a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}} - \frac{(x+a)}{\{(x+a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$
$$= \left(\frac{l}{4\pi}\right) \left(\frac{z}{y^{2} + z^{2}}\right) \left(\frac{-2a}{\{x^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$
$$H_{z} = \left(\frac{l}{4\pi}\right) \left(\frac{-y}{y^{2} + z^{2}}\right) \left(\frac{(x-a)}{\{(x-a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}} - \frac{(x+a)}{\{(x+a)^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$
$$= \left(\frac{l}{4\pi}\right) \left(\frac{-y}{y^{2} + z^{2}}\right) \left(\frac{-2a}{\{x^{2} + y^{2} + z^{2}\}^{\frac{1}{2}}}\right)$$



Compared with task 1 the field components vary in a much steeper way in this example. One could view this as the field decaying much easier at distances away from the wire but at the same time close to the wire the field peaks faster because it carries the same current at a reduced length. In task 1 the field components were inversely proportional to y and z and here it seems that the magnitudes of the components are inversely proportional to the square of the coordinates with the direction of the field being towards the wire.



The matlab results closely follow the analytical expression by plotting both variations in Excel.

The additional lines of code are shown below: **case 4**



The field components on the yz observation plane closely follow the behaviour of the components of example 2.

We want to compare the z component of the field along the z axis with the the z component of a circular loop of radius R given by : $H_z = IR^2/\{2(Z^2 + R^2)^{3/2}\}$ and we plot the log log plot of the two field components below:



The above means that we can approximate a circular loop using a square current carrying loop which is much easier to implement using matlab.

The extra lines of code are shown below: case 5



For the z component along the y axis, the field resembles task 2. Indeed what is happening here is that the axis is divided in segments where for each segment 2 wires have dominant fields that superimpose. For -50 < y < -27.5 the dominant wire is the leftmost wire. For -27.5 < y < -2.5 the dominant wires are the parallel wires along the x axis of the leftmost square loop giving a field component quite similar with task2. Symmetrically, this effect is the same for the rest of the segments of y axis. For the field component along the z axis, we observe a peak on z=0 where the distance from the square loops is minimised.



In comparison with example 5 one could draw the conclusion that when the two loops carry current in the same direction, the magnetic field created is evenly symmetrical whereas in example 6 where the 2 loops carry current at opposite directions, the symmetry of the magnetic field is odd which is clearly depicted in the graphs. Apart from the even and odd symmetries of example 5 and 6 respectively, the magnitudes and behaviour of the curves is exactly the same.

```
Task 7
case 7
    NW = 35;
     I = 1/NW;
     W = 10;
     S = 2;
     central Wire = [
         0,W/2,0,3,W,I;
         0, -W/2, 0, 3, W, -I;
         0, 0, W/2, 2, W, -I;
         0,0,-W/2,2,W,I
         ];
     WIRE= [central_Wire];
     for i = 1:(NW-1)/2
         wire_positive = central_Wire;
         wire_negative = central_Wire;
         wire_positive(:,1) = central_Wire(:,1) + i*S;
         wire_negative(:,1) = central_Wire(:,1) - i*S;
         WIRE = [WIRE;wire_negative];
         WIRE = [WIRE;wire_positive];
     end
```

The lines of code above are needed to produce the solenoid containing NW turns. Initially, we create a central wire with width 10 carrying a current of 1/NW. Then we need a for loop that does the following:

At each iteration 2 wires are added to the configuration. One that is right of the central wire with separation of i*S and a second one that is one the left of the central wire with separation



After running the code with observation plane on the xy plane we get the results above.



An important thing to note is that the x field component inside the solenoid is constant. This
can be verified by the field lines which are mostly parallel. Also note that the y and z
components are 0.

NW	H_X
15	31.2
25	19.5
35	14.1
45	11
55	9.04
66	7.66

To observe the relationship of NW to the x component of the field, we run the code for various values of NW and measure the constant value of the x component of the magnetic field inside the solenoid. The overall magnitude drops as NW increases. One should consider that the current across the wire remains constant throughout measurements so the length of the solenoid increases but the current per unit length decreases. This most probably is the reason behind the drop of the magnitude of the field inside the solenoid. Also note how outside of the solenoid the magnitude of the x component decays to 0. The y and z components of the field have magnitudes outside of the solenoid. The field graphs do not show that magnitude because for the central lines indeed these components have zero magnitude. If someone observed the field lines though there are points in the plane for example(x=10,y=10) where there is a y component of the field. Using an observation plane at the xz plane we get:



The behaviour is exactly the same as in the xy plane but for the z component of the field.

```
case 8
   SY = 0.5;
   SZ = 4;
   NW = 25;
   I = 1/NW;
   L=2000:
   central_Wire_positive = [0,0,SZ/2,1,L,I];
   central_Wire_negative = [0,0,-SZ/2,1,L,-I];
   WIRE = [central_Wire_negative;central_Wire_positive];
    for i = -(NW-1)/2:-1
       wire_positive = central_Wire_positive;
       wire_negative = central_Wire_negative;
       wire_positive(:,2) = central_Wire_positive(:,2) + i*SY;
       wire negative(:,2) = central Wire negative(:,2) + i*SY;
       WIRE = [WIRE;wire_negative;wire_positive];
   end
    for i = 1:(NW-1)/2
       wire positive = central Wire positive;
       wire_negative = central_Wire_negative;
       wire_positive(:,2) = central_Wire_positive(:,2) + i*SY;
       wire_negative(:,2) = central_Wire_negative(:,2) + i*SY;
       WIRE = [WIRE;wire_negative;wire_positive];
    end
```

The code above is simple. I used 2 for loops to divide the task at hand, maybe 1 for loop could be used as well to avoid redundancy. Initially we have two wires at the center (y = 0). The first one has a z coordinate at SZ/2 and the second one has a z coordinate at -SZ/2 so that there is a separation of SZ. Also the top wire carries current I and the second wire carries current -I. The next for loop places (NW-1)/2 pairs of wires at the left of the central wire and the second for loop places the same amount of wires on the right of the central wire.



The result above verifies that the code worked correctly.



The field lines seem to have a faint similarity with task 7 but note that this arrangement is not a solenoid.



The field components have an expected behaviour. For values of y outside of the current strips the field components decay to 0. For the field component along the z axis the magnitude is 0 and peaks negatively on top of the current strips while also peaking at a magnitude between the strips on the positive direction.